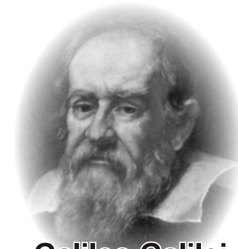
Pendulum Motion

The Value of Idealization in Science



Galileo Galilei

The pendulum is seemingly a very humble and simple device. Today it is mostly seen as an oscillating weight on old grandfather clocks or as a swinging weight on the end of a string used in school physics experiments. But surprisingly, despite its modest appearances, the pendulum has played a significant role in the development of Western science, culture and society.

The significance of the pendulum to the development of modern science was in its use for time keeping. The pendulum's initial Galileo-inspired utilization in clockwork provided the world's first accurate measure of time. The accuracy of mechanical clocks went, in the space of a couple of decades in the early 17^{th} century, from plus or minus half-an-hour per day to one second per day. This quantum increase in accuracy of timing enabled previously unimagined degrees of precision measurement in mechanics, astronomy and other fields of study. It ushered in the world of precision so characteristic of the scientific revolution. Time could then confidently, and for the first time, be expressed as an independent variable in the investigation of nature. For example, each of the following could, for the first time, be reliably investigated:

- the effect of force on objects over time,
- the distance of fall over time,
- the change of speed over time,
- the radial movement of planets over time, and
- the progress of chemical reactions over time.

All of these investigations required that time accurately and reliably be measured. Competent time measurement was a requirement for modern science and the pendulum enabled this to happen.

The pendulum was studied by Galileo, Huygens, Newton, Hooke and all the leading figures of the seventeenth-century Scientific Revolution. Many people are aware that the great Isaac Newton was a pivotal figure in the creation of modern science. Some even know that his major work was a book published in 1687 titled *Principia Mathematica*. But most people, physicists included, would be surprised to learn that Newton built his new physics and astronomy on his work regarding the properties of pendulum motion.

The pendulum played more than a scientific and technical role in the formation of the modern world. It also indirectly changed cultures and societies through its impact on navigation. Accurate time measurement was long seen as the solution to the problem of longitude determination which had vexed European maritime nations in their efforts to sail beyond Europe's shores. Treasure fleets from Latin America, trading ships from the Far East, and naval vessels were all getting lost and running out of food and water. Ships were running aground on reefs or into cliffs instead of finding safe harbors.

Position on the Earth's surface is given by two coordinates: *latitude* (how many degrees above or below the equator the place is), and *longitude* (how many degrees east or west of a given north-south meridian the place is). When traveling over land or sailing across oceans, travelers must know the coordinates (latitude and longitude) of their destination, the coordinates of their present position, and some way of guiding themselves between those positions. Knowledge of latitude and longitude is essential for accurate map making. Losing one's way on the ocean frequently amounted to 'being lost at sea'. Without knowledge of position, reliable traveling and trading was problematic and dangerous.

Determination of latitude was a relatively straightforward matter. The inclination of the pole star (when north of the equator), inclination of the sun (when either north or south of the equator), and length of day had all been used with moderate success to ascertain how many degrees above or below the equator the observer was. But for two thousand years, finding or specifying longitude was an intractable problem. Governments and kings offered huge rewards for anyone who could solve this vexing problem.

The various Royal prizes, and particularly the British Longitude Act, spawned a host of eccentric would-be solutions to longitude. But the ultimately correct and workable method had been first proposed by Gemma Frisius (1508-1555), the Flemish astronomer, professor at Louvain University, and teacher of Mercator the map maker. In 1530 he proposed time-keeping as the solution to the problem of longitude.

The science behind Frisius' suggestion is that the Earth makes one revolution of 360° in 24 hours. Thus, in one hour it rotates through 15°, or 1° every four minutes.

Although the line of zero longitude is arbitrary, an exact relationship between time and longitude exists. Say a clock was set to accurately read 12 o'clock at noon (that is, when the sun is highest in the sky) at the outset of the journey. Provided the clock kept time reliably, one only had to look at the clock when the sun was at its highest (noon) throughout the journey in order to find out how many degrees east or west of the beginning one was. If the clock read 2 o'clock when the sun was highest then you were 30° west of the starting point. If the clock read 9 o'clock when the sun was highest, then you were 45° east of where you started.

Gemma Frisius

If an accurate and reliable clock was carried on voyages from London, Lisbon, Genoa, or any other port, then by comparing its time with local noon (as determined by the sun's shadow), the longitude of any place in the journey could be ascertained. As latitude could already be determined, this enabled the world to be mapped. In turn, this provided a firm base on which European exploitation, colonization and commerce could proceed.

The big problem was that there were no accurate clocks! The Earth's circumference at the equator is about 40,000 km. So one degree of longitude at the equator is 40,000km/360 or about 110 kms per degree longitude. As the earth rotates on its axis each 24 hours (1440 minutes), one minute of rotation time at the equator is equal to about 28 km. As the best mechanical clocks were accurate only to about 30 minutes per day, the positional error could be 830 km! Even with considerable increase in timekeeping accuracy, a perilous margin of error still existed. A clock that gained or lost even five minutes per day meant that position at sea, in the equatorial regions, could be ascertained to only about plus or minus 140 kms. This gives a lot of room to run into a reef, an island, or a cliff face at night. And of course, these calculations assume that the sailor could reset their clock each local noon. If the sky was cloudy or the boat was severely pitching, then ascertaining local noon was problematic or perhaps impossible.

For over thirty years, Galileo was an energetic participant in the quest for the solution to the longitude problem. Accurate timekeeping was recognized as the solution to the longitude problem, and the pendulum played a pivotal role in solving it. Galileo and most of the famous scientists of the seventeenth century worked intimately with clockmakers, and used their analysis of pendulum motion (specifically, the insight that for a given pendulum length, all periods are the same) in the creation of qualitatively more accurate clocks.

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The study of pendulum motion illustrates how science may be influenced by social and economic problems, and how science can in turn impact society. This mutual impact is not unusual, and is illustrated throughout historical and contemporary science and society.

In a letter of 1632, ten years before his death, Galileo surveyed his achievements in physics and recorded his debt to the pendulum for enabling him to measure the time of free-fall. He wrote that, "we shall obtain from the marvellous property of the pendulum, which is that it makes all its vibrations, large or small, in equal times".

Galileo at different stages in his work makes four novel claims about pendulum motion:

1) Period varies with length and later, more specifically, the square root of length; the Law of Length.

2) Period is independent of amplitude; the Law of Amplitude Independence.

3) Period is independent of weight; the Law of Weight Independence.

4) For a given length, all periods are the same; the Law of Isochrony.

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Scientific laws are statements of invariable relationships. Scientific theories explain why those relationships exist. Both have an inventive character. As you read below, consider how scientific laws do not follow simply from observation.

Galileo's laws are taught in all high school and university physics programs, with the topic frequently being taught as if they were self-evident. But something is a bit peculiar here. Despite people seeing swinging pendulums for thousands of years, no one, not even the great Leonardo da Vinci who studied pendulum motion, noted what Galileo "saw." In the Western world, none of the great philosophers and investigators of nature from ancient times through to the seventeenth century had noticed these properties of the pendulum; nor it seems did anyone in the great civilizations of China, India or the Middle East. Some knowledge of the rich history behind Galileo's pendulum work helps in understanding the physics of pendulum motion. The story also illustrates one of the defining methodological features of the Scientific Revolution and of modern science to be appreciated, namely the importance of idealization in the revolutionary scientific achievements of Galileo and Newton.

Since the fourth century BC, Aristotle's emphasis on ordinary experience had held sway in efforts to comprehend natural phenomena. The emphasis on commonplace observation made sense, and likely makes sense to most people today. However, the seventeenth century's analysis of pendulum motion illustrates a different way of thinking that is the methodological heart of the Scientific Revolution. More particularly, the debate between the Aristotelian Guidobaldo del Monte and Galileo over the latter's claims about pendulum motion, represents, in microcosm, the larger methodological struggle between Aristotelianism and the new science. This struggle is in large part about the legitimacy of idealization in science, and the utilization of mathematics in the construction and interpretation of experiments.

1. Aristotle emphasized that science should be based on ordinary observation of phenomena, and most people agree. In fact, science teachers and curricular materials often convey this notion. Why does the idea that science should be based solely on what we see make so much sense?

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As you read further about Galileo's insights on pendulum motion, consider how the prevailing Aristotelian emphasis on ordinary observation had to be reconsidered.

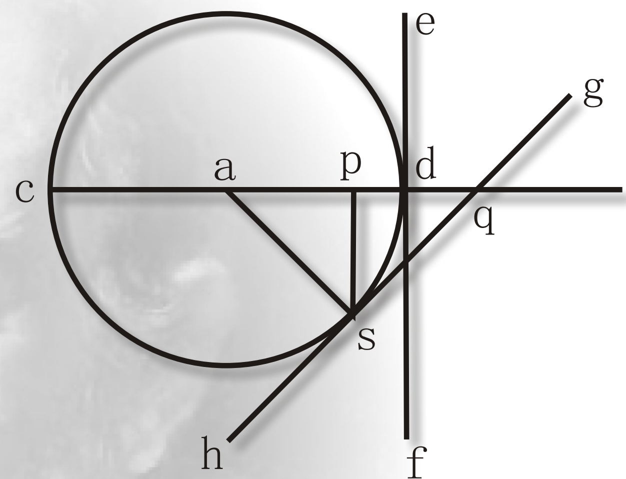
The most significant opponent of Galileo's emerging views about the pendulum was his own academic patron, the distinguished Aristotelian Guidobaldo del Monte (1545-1607). Del Monte was one of the great mathematicians and mechanicians of the late sixteenth-century, a highly competent mechanical engineer and Director of the Venice Arsenal. Additionally he was an accomplished artist, a minor noble, and the brother of a prominent cardinal of the Catholic Church. Further he secured for Galileo his first university position as a lecturer in mathematics at Pisa University (1588-1592), and his second academic position as a lecturer in mathematics at Padua University (1592-1610).

Del Monte was not only a patron of Galileo, but from at least 1588 to his death in 1607, he actively engaged in Galileo's mechanical and technical investigations. They exchanged many letters and manuscripts on problems of mechanics and natural philosophy. Galileo sent del Monte his mathematical proof for the law of isochronous pendulum motion. It was a geometrical construction as follows.

Figure 1 depicts a pendulum suspended at A and swinging along the lower half of the circle D, S through to C. Galileo argued that whether the pendulum was released from D or from S, or even lower on the circle, its period would be the same, and further *each* of its swings would take the same time.

FIGURE 1

Galileo's explanation that all pendulums swing with equal periods.



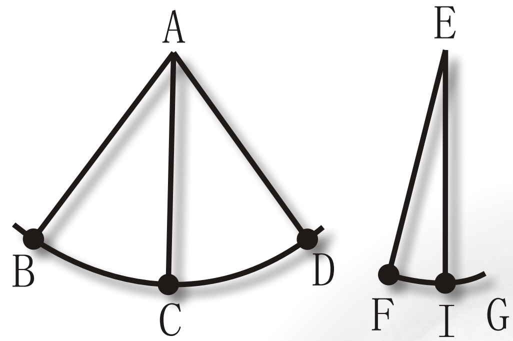
In one famous letter to del Monte written in 1602, Galileo wrote:

> Therefore take two slender threads of equal length, each being two or three braccia long [four to six feet]; let these be AB and EF (Figure 2). Hang A and E from two nails, and at the other ends tie two equal balls (though it makes no difference if they are unequal). Then moving both threads from the vertical, one of them very much as through the arc CB, and the other very little as through the arc IF, set them free at the same moment of time. One will begin to describe large arcs like BCD while the other describes small ones like. Yet in this way the moveable [that is, movable body] B will not consume more time passing the whole arc BCD than that used up by the other moveable F in passing the arc. I am made quite certain of this.

All of this looked terrific, and the mathematics was great. But del Monte kept saying that real pendula do not behave as Galileo's mathematical proofs assert. He said that Galileo was an admirable mathematician and that is why he recommended Galileo for his first two positions as a professor of mathematics at Pisa and Padua universities. Del Monte maintained that Galileo was an excellent mathematician but a very poor physicist. Mathematics, he argued, tells us about an abstract unreal world, while physics (natural philosophy) has to tell us how the real world actually behaves.

FIGURE 2

Galileo's comparison of the periods of two pendulums.



Del Monte repeated many of Galileo's claimed experiments and demonstrations and noted that they did not work. Galileo said that heavy and light pendula stay in perfect synchrony for 'thousands of oscillations, yet del Monte noted that they were out of synchrony in just a dozen complete swings. When del Monte released balls on the inside of iron cylinders, the one released from just 2 degrees got to the lowest point well before the one released from 45 degrees. Moreover, the claim of isochronous motion for the pendulum meant that every swing takes the same time as the first swing. This implied perpetual motion and pendula manifestly are not perpetual mobiles. Even the most sophisticated pendulum when freely released ceases swinging in about 100 oscillations.

2. What do Aristotelians consider the purpose of science to be, and how is Del Monte using this view to oppose Galileo's claims about pendulum motion?

Del Monte thought that theory should not be separated from application, that mind and hand should be connected. As he wrote in the Preface of his *Mechanics*: "For mechanics, if it is abstracted and separated from the machines, cannot even be called mechanics". Del Monte was concerned with the long-standing Aristotelian problem of how mathematics related to physics. In his *Mechaniche* he writes:

> Thus, there are found some keen mathematicians of our time who assert that mechanics may be considered either mathematically, removed [from physical considerations], or else physically. As if, at any time, mechanics could be considered apart from either geometrical demonstrations or actual motion! Surely when that distinction is made, it seems to me (to deal gently with them) that all they accomplish by putting themselves forth alternately as physicists and as mathematicians is simply that they fall between stools, as the saying goes. For mechanics can no longer be called mechanics when it is abstracted and separated from machines. (Drake & Drabkin 1969, p.245)

The methodological divide between del Monte and Galileo, between Aristotelian science and the embryonic new science of the Scientific Revolution, was signaled in del Monte's criticism of contemporary work on the balance, including perhaps drafts of Galileo's first published work, *La Bilancetta* in 1586. Del Monte cautioned that physicists are:

> … deceived when they undertake to investigate the balance in a purely mathematical way, its theory being actually mechanical; nor can they reason successfully without the true movement of the balance and without its weights, these being completely physical things, neglecting which they simply cannot arrive at the true cause of events that take place with regard to the balance. (Drake & Drabkin 1969, p. 278)

This then is the methodological basis for del Monte's criticism of Galileo's mathematical (or geometric) treatment of pendulum motion. Del Monte, a mathematician and a great technician, is committed to the core Aristotelian principle that physics, or science more generally, is about the world as experienced, and that sensory evidence is the bar at which alleged physical principles are examined. Vision was Aristotle's primary sense; it provided the material for mind. In his *On the Generation of Animals*, Aristotle remarks:

> Credit must be given rather to observation than to theories, and to theories only if what they affirm agrees with observed facts. (Barnes 1984, p.1178)

This appeal to observation is also the commonsense understanding of science held then, and today by most people. The shift to using mathematics to understand the natural world is the methodological core of the Scientific Revolution. The subsequent development of pendular analyses by Huygens, and then Newton, beautifully illustrate the interplay between mathematics and experiment so characteristic of the emerging Galilean-Newtonian Paradigm.

In his more candid moments, Galileo acknowledged that events do not always correspond to his laws; that the material world and his so-called 'world on paper', the mathematical world, did not correspond. But Galileo maintained that *accidents* interfered with del Monte's tests: his wheel rim was not perfectly circular, it was not smooth enough, the balls were subject to friction, and so on.

Immediately after mathematically establishing another of his famous relationships the law of parabolic motion of projectiles -Galileo remarks that:

> I grant that these conclusions proved in the abstract will be different when applied in the concrete and will be fallacious to this extent, that neither will the horizontal motion be uniform nor the natural acceleration be in the ratio assumed, nor the path of the projectile a parabola. (Galileo 1638/1954, p. 251)

Note that Galileo is arguing that under idealized conditions (e.g. no friction, mass that takes up no space) pendula would behave as his mathematics illustrates. So his laws of pendulum motion are universal – they apply everywhere and any observed deviation can be predicted by noting how far real conditions deviate from the ideal.

One can imagine the reaction of del Monte and other hardworking Aristotelian natural philosophers and mechanicians when presented with such a qualification. When baldly stated, it was directly at odds with the basic Aristotelian and empiricist objective of science, namely to tell us about the world in which we live. Consider, for instance, the surprise of Giovanni Renieri, a gunner who attempted to apply Galileo's parabolic theory to his craft. When he complained in 1647 to Torricelli that his guns did not behave according to Galileo's predictions, he was told by Torricelli that 'his teacher spoke the language of geometry and was not bound by any empirical result' (Segre, 1991, p. 43).

Galileo is not deterred by the 'perturbations', 'accidents', and 'impediments' that interfere with the behavior of the free falling, rolling, and projected bodies with which his New Science is dealing. His procedure is explicitly stated immediately after the disclaimer about the behavior of real projectiles in contrast to his ideal ones. Galileo in 1638 wrote:

> Of these properties [*accidenti*] of weight, of velocity, and also of form [*figura*] infinite in number, it is not possible to give any exact description; hence, in order to handle this matter in a scientific way, it is necessary to cut loose from these difficulties; and having discovered and demonstrated the theorems, in the case of no resistance, to use them and apply them with such limitations as experience will teach.

In an historical understatement, Galileo added: "And the advantage of this method will not be small."

3. What is gained and lost by stating scientific laws (i.e., invariable relationships) that are dependent upon idealized conditions?

Modern science is based on the Galilean-Newtonian framework that had to be established against Aristotelian opposition that was committed to the commonsense view that science was supposed to tell us about the world that we immediately experience. Modern philosophers understand this. Michael Scriven arrestingly remarked that 'The most interesting thing about laws of nature is that they are virtually all known to be in error' (Scriven 1961, p. 91). Nancy Cartwright in her *How the Laws of Physics Lie* says that if the laws of physics are interpreted as empirical, or phenomenal, generalizations, then the laws lie (Cartwright 1983). Cartwright states the matter as: 'My basic view is that fundamental equations do not govern objects in reality; they govern only objects in models' (Cartwright 1983, p. 129). The world does not behave as the fundamental equations dictate. This claim is not so scandalous: the gentle and random fall of an autumn leaf obeys the law of gravitational attraction, but its actual path is hardly as described by the equation $s = \frac{1}{2} g t^2$. This equation refers to idealized situations. A true description, a phenomenological statement, of the falling autumn leaf would be complex beyond measure. The mathematical law of falling bodies states an *idealization*, but an idealization that can be experimentally approached.

4. Why do idealizations have to be made in science? If scientific laws do not literally describe natural phenomena, how do they relate to the world?

Galileo was convinced that a perfect pendulum swinging in a circular path in an 'accident-free' situation would keep perfect time; its beats would be regular and synchronous. Towards the end of his life he drew up plans for the construction of a pendulum clock based on this principle which he thought would solve the longitude problem and be of the greatest value to science. Unfortunately he died before his clock could be constructed. But the idea was taken over by the great Dutch scientist Christiaan Huygens who published the *Horologium Oscillatorium* ('The Pendulum Clock') in 1673. Working with excellent technicians he oversaw the making of the world's first pendulum clock that was accurate to one minute per day, and shortly after to one second per day. This then launched the era of precision timekeeping that enabled Western science to rapidly progress. It also enabled the longitude problem to be solved, facilitating global exploration, trading and conquest by the European maritime powers.

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